1906MNRAS..67...17C

observations of the Sun cannot be said to be inconsistent with the extended hypothesis.

The evidence that has been advanced serves to show that the facts brought forward by Mr Cowell in support of his views can be explained without having recourse to a new secular acceleration in the mean motion of the Earth, so that these facts do not suffice to establish the existence of such a secular acceleration. But if this is all, they certainly do not suffice to disprove the existence of such a term. If that is to be done, it must be by further direct evidence, to which I have no access, though it may exist. During past years in my researches I have repeatedly found evidence of an unexplained apparent secular acceleration in the motion of the Moon's argument of latitude, but I have doubted its reality, as no explanation of its origin seemed available. The explanation advanced by Mr Cowell, that it might be due to a secular acceleration in the mean motion of the Earth, did not occur to me, or I should have taken the necessary steps to obtain the data to investigate the matter. This, I presume, will be done by Mr Cowell. I do not see any theoretical explanation of such a secular acceleration which does not seem to involve great difficulties. Mr Cowell's tentative explanation (*Monthly Notices*, vol. lxvi. p. 352) I think he will find untenable on review.

Natal Observatory:
1906 *August* 23.

The Mediæval Eclipses of Celoria. By P. H. Cowell.

Professor Celoria has collected a number of references to the total eclipses of 1239 June 3 and 1241 October 6. He considers that the former eclipse was certainly total at Piacenza and at Lesina, and that the latter was total at Stadt and at Ellwangen. He draws the limits of totality according to Hansen's Tables (without Newcomb's corrections), and concludes that these tables are in serious error at the epoch of the two eclipses.

Professor Celoria's diagrams show that the conditions that he imposes are very stringent ones. The zone of totality for the 1239 eclipse is to be displaced not less than enough to secure totality at Piacenza, and not so much as to destroy totality at Lesina, and these limits are very narrow. They become still narrower if it be remembered that Hansen's semi-diameter is considerably too large for eclipse purposes. A similar remark applies to the eclipse of 1241. Professor Celoria's assumptions, therefore, lead to two equations of condition, one for each eclipse, in which the margin of uncertainty is very slight. The combination of the two should therefore give an excellent determination of the corrections required by the mean elongation and by the argument of latitude.

The form of the equations of condition can be most simply found. Let k denote, as in my previous papers on eclipses, the

1906MNRAS..67...17C

ratio of the Moon's motion in latitude to the difference of motion in longitude as seen by the observer, and let $\delta(V-V')$ and δU be the corrections required by the difference of longitude and by the Moon's latitude. Then the equation of condition must be

$$\delta U - k\delta(V-V') = a$$

where a is a constant, that depends upon the tables to be corrected.

That this is the true equation of condition is most easily seen; for if δU, $\delta(V-V')$ be one solution, $\delta U + k\delta t$, $\delta(V-V') + \delta t$ must be another solution, corresponding to a small change in the assumed time at which the eclipse occurred.

For any place on the Earth's axis, including the centre of the Earth and the two poles, the parallactic displacements are constants, and the value of k will be $\pm 0{\cdot}10$, the upper sign corresponding to the ascending node. For moderate latitudes k is approximately

$$\pm\ 0{\cdot}14 + 0{\cdot}14 \cos h$$

where h is the sidereal time.

Of course this formula is only a rough one, but it serves the purpose, before undertaking laborious calculations, of predicting that the equations of condition will involve

$$\delta U - 0{\cdot}12\ \delta(V-V') \text{ for the 1239 eclipse}$$
$$\text{and } \delta U + 0{\cdot}26\ \delta(V-V') \text{ for the 1241 eclipse.}$$

Writing
$$\delta U = \pm 0{\cdot}09\ \delta F$$
$$\delta\ (V-V') = \delta D$$

the unknown corrections to the argument of latitude and mean elongation enter in the form

$$0{\cdot}09\ \delta F - 0{\cdot}12\ \delta D$$
and
$$-0{\cdot}09\ \delta F + 0{\cdot}26\ \delta D$$

We shall not therefore have any trouble arising from the determinant of the coefficients being small.

Professor Celoria has obtained the equations

$$9{\cdot}8635\ \delta v - 5{\cdot}4914\ \delta\Omega = 35'{\cdot}40$$
$$-57{\cdot}7624\ \delta v - 1{\cdot}4098\ \delta\Omega = -42{\cdot}60$$

These equations cannot be reconciled with the form of mine, which has been justified by such elementary reasoning.

I obtain the following expressions for the difference of apparent latitude at the instant of apparent conjunction in longitude, δF, δD being corrections to the values of F, D that correspond to the formulæ in *Monthly Notices*, vol. lxvi., p. 525.

1239, for Piacenza	$0{\cdot}13\delta D - 0{\cdot}09\delta F + 89''$
1239, for Lesina	$0{\cdot}11\delta D - 0{\cdot}09\delta F - 47''$
1241, for Stadt	$-0{\cdot}25\delta D + 0{\cdot}09\delta F + 29''$
1241, for Ellwangen	$-0{\cdot}27\delta D + 0{\cdot}09\delta F - 57''$

1906MNRAS..67...17C

Hence, in order to produce totality at all four places, we must have

$$0{\cdot}12\delta D - 0{\cdot}09\delta F = -21''$$

and

$$-0{\cdot}26\delta D + 0{\cdot}09\delta F = +14''$$

with errors of not more than three or four seconds on the right-hand sides.

The solution of these equations is $\delta D = 50''$, $\delta F = +300''$.

The value of δD is a possible one, but the value of δF is utterly impossible. It cannot possibly be reconciled with modern observations. Matters are even worse if the comparison be made with the present tables. The latter require an even larger correction δF.

I interpret allusions to darkness and to the appearance of stars to mean that the place of observation was not necessarily within the zone of totality, but within a somewhat wider zone, within which not more than about twenty seconds of the Sun's diameter was visible.

Now this explanation not only serves to dispose of an inconveniently large correction to the formulæ used in calculation, but it is absolutely required by the records. For at Altenzelle in 1241 we have the record " . . . . et stellæ apparuerunt," and at Vienna we have two records to the same effect. Professor Celoria confines the places to which he assigns black rings on his maps to within a zone not wider than the zone of totality. As the distance between Ellwangen and Vienna is about as great, when projected perpendicularly to the central line, as the greatest width that can be ascribed to the zone within which stars appeared, the central line probably was about equidistant from the two places. This fits my formulæ well, but I cannot use the principle to discriminate between my formulæ and the present tables.

In 1239 the record at Milan has to be rejected, as Professor Celoria suggests, on the ground that the writer lived 160 years later. We are then left with the records that stars were seen at Piacenza, and that the eclipse was certainly total at Lesina.

To sum up, therefore, my formula cannot be accepted unless it be admitted that at Piacenza in 1239 stars might have been seen with about 20″ of the Sun's diameter visible; on the other hand, if the theoretical secular acceleration be attributed to the argument of latitude, 30″ of the Sun's diameter would have been visible. I am not prepared to say that 20″ is possible and 30″ impossible, and therefore no proof can be based upon these eclipses. I feel bound, however, to examine all evidence that I come across, and I can only say that I am perfectly satisfied with the way in which my formulæ stand the test in the present instances.

The outline of the calculations is as follows:—

Changes in a Julian century $\times 10^{-7}$.

	1239	1241
	″	″
True Longitude of Moon .	198·64	199·72
,, ,, Sun .	12·53	13·12
True Elongation . . .	186·11	186·60

1906MNRAS..67...17C

	1239 Piacenza.	1239 Lesina.	1241 Stadt.	1241 Ellwangen.
Parallax in Longitude .	55·3	55·2	45·0	49·5
Apparent Elongation .	130·8	130·9	141·6	137·1
Geocentric Latitude of Moon	+18·42		−18·24	
Parallax in Latitude	− 1·2	− 4·2	−17·7	−19·6
Apparent Latitude of Moon . . .	+17·2	+14·2	−35·9	−37·8
Hence *k* . . .	+ 0·13	+ 0·11	− 0·25	− 0·27

	1239	1241
T in centuries from 1800 Jan. 0·06 M T	−5·6054754	−5·5820399
G.M.T. in degrees . . .	+3°·9654	−2°·6451

	1239 °	′	″	1241 °	′	″
F . . .	5	23	36·4	169	28	27·5
D . . .	2	0	1·6	357	3	26·9
g . . .	344	26	51·5	7	50	14·0
g′ . . .	168	39	16·1	292	18	35·7
−Ω . . .	284	51	39·0	330	11	20·6

	1239	1241
Inequalities of Moon's Longitude :—	″	″
α. 21 largest terms, coefficients over 20″	−5096·2	+3299·2
β. 30 next largest solar terms, coefficients over 1″·35 . . .	− 5·9	+ 53·5
γ. 58 next largest solar terms, coefficients over 0″·15 . . .	+ 4·0	− 0·4
δ. Figure of Earth terms . . .	+ 7·1	+ 3·2
Inequalities of Moon's Latitude :—		
α. 14 largest terms, coefficients over 10″	+1271·3	+3118·0
β. 42 next largest solar terms, coefficients over 0″·55 . . .	− 21·1	− 63·7
γ. 62 next largest solar terms, coefficients over 0″·06 . . .	− 0·4	− 3·9
δ. Figure of Earth terms . . .	− 7·5	+ 1·2
Moon's sine parallax. Constant, 3422″·7 :—		
α. 5 largest terms, coefficients over 10″	+3671·5	+3678·7
β. 9 next largest solar terms, coefficients over 0″·55 . . .	− 0·9	+ 4·2
γ. 17 next largest solar terms, coefficients over 0″·06 . . .	+ 1·0	− 0·2
Inequalities of Sun's Longitude :—		
Three terms of equation of centre	+22′ 33″·1	−1° 49′ 7″·7

	Piacenza.	Lesina.	Stadt.	Ellwangen.
Geocentric Latitude	44°51′·0	42°58′·6	53°24′·6	48°45′·6
Radius Vector *ρ* .	·99830	·99841	·99779	·99807
Longitude, East of Greenwich . .	9°41′·55	16°28′·22	9°28′·43	10°9′·92
Parallax in Longitude :—	″	″	″	″
α. Terms of first order	− 740·3	−1055·0	+ 850·1	+ 726·7
β. „ second „	− 11·9	− 16·8	+ 7·1	+ 6·9
γ. „ third „	− 0·2	− 0·2	+ 0·2	+ 0·2

1906MNRAS..67...17C

Parallax in Latitude :—

α. Terms of first order $-1329{\cdot}8 \quad -1230{\cdot}1 \quad -3122{\cdot}2 \quad -3006{\cdot}7$

β. Smaller terms $\quad - \quad 1{\cdot}4 \quad + \quad 0{\cdot}2 \quad - \quad 0{\cdot}7 \quad + \quad 0{\cdot}3$

The formulæ used for the arguments are those given in *M. N.*, lxvi. p. 525.

The inequalities of the Moon's Longitude, Latitude, and Parallax are taken from Brown, *M. N.*, lxv. p. 276.

The inequalities of the Sun's Longitude and the obliquity of the ecliptic are taken from Newcomb, Tables of the Sun.

The figure of Earth's terms used are :—

For Moon's Longitude $+ 6''{\cdot}6 \sin \Omega - 0''{\cdot}7 \cos \Omega + 1''{\cdot}0 \sin \Omega \cos$

For Moon's Latitude $- 8''{\cdot}1 \sin \mathrm{L} + 1''{\cdot}5 \cos \mathrm{L} - 1''{\cdot}0 \sin g \cos \mathrm{L} - 0''{\cdot}3 \sin (\mathrm{F} - \Omega)$

Nutation is not applied, since it is the same for both Sun and Moon, and not sufficiently important in its effect on the parallax.

The planetary inequalities have not been calculated for either Sun or Moon.

The formulæ used for parallax are

$$q = \rho\ (\sin p - p')$$

Parallax in longitude $- bq - abq^2 + (\tfrac{1}{3}b^3 - a^2 b)q^3 - \tfrac{1}{2}bq\mathrm{U}^2$

„ latitude $- cq + aq(\mathrm{U} - cq) - \tfrac{1}{3}\mathrm{U}^3$

where $a = \quad \cos\lambda \cos h \cos \mathrm{V} + \cos\lambda \sin h \cos\epsilon \sin \mathrm{V} + \sin\lambda \sin\epsilon \sin \mathrm{V}$

$b = -\cos\lambda \cos h \sin \mathrm{V} + \cos\lambda \sin h \cos\epsilon \cos \mathrm{V} + \sin\lambda \sin\epsilon \cos \mathrm{V}$

$c = \qquad -\cos\lambda \sin h \sin\epsilon + \sin\lambda \cos\epsilon$

Less exact calculations would have been sufficient if I could have foreseen that the only result is that my formulæ expose less of the Sun's disc at the places named as places where stars appeared than the theoretical formulæ.

The present paper is most easily read in connection with Professor Celoria's diagrams. These, however, can be sufficiently reproduced from the following extracts from Professor Celoria's papers. The latitudes are geographical.

1239 June 3. Limits of Totality.

North		South	
43° 6′·84 N	0° 29′·79 E	40° 49′·52 N	1° 12′·58 E
44 18 ·97	8 23 ·34	41 58 ·58	8 33 ·51
44 43 ·80	17 1 ·14	42 24 ·64	16 34 ·23

1241 October 6. Limits of Totality.

North		South	
58° 35′·96 N	6° 42′·01 E	55° 56′·48 N	4° 59′·05 E
51 57 ·27	12 48 ·14	49 41 ·56	10 49 ·69
46 37 ·08	17 24 ·99	44 34 ·64	15 20 ·49

and

Place.	Latitude.	Longitude.
Milan	45° 27′ 59″	9° 11′ 14″
Altenzelle	51 3′·55	13 16′·30
Vienna	48 12 ·55	16 22 ·52

1906MNRAS..67...17C

The positions of the other places mentioned in this paper are given above.

Professor Celoria's extracts from mediæval records represent a vast amount of valuable labour. It is advisable, however, to warn readers that his figures for the 1239 eclipse must not be taken from his original paper, but from his own revised figures in his second paper.

The Irregular Movement of the Earth's Axis of Rotation: a Contribution towards the Analysis of its Causes. By Professor J. Larmor, F.R.S., and Major E. H. Hills, C.M.G.*

Much material, defining with increasing accuracy the irregular wanderings of the Earth's axis of rotation, has now been accumulating for a long series of years. The attempt to decompose the movement into regular harmonic components excited interest some six or eight years ago. Since that time the more systematic data obtained and analysed by the International Organisation have given greater precision to the path of the Pole; and, while a definite astronomical discussion must rest with the experts, curiosity as to the general physical causes of the phenomenon is legitimate. It has long been recognised that displacement of material on the Earth's surface due to meteorological changes (melting of polar ice, long-period barometric fluctuations, etc.) must be a prominent agent, and may indeed be taken to be the main one; while Newcomb has pointed out that the free Eulerian oscillatory period must be very different, for a nearly spherically balanced Earth, if it is elastically deformable under the centrifugal force, from what it would be if it were rigid, thus accounting for the unexpected value of the Chandler period.

It is shown below that, without making any hypothesis except the natural one, that this free precessional period is fixed in duration and determines the average duration of the revolutions of the Pole of rotation, it is easy by a graphical process to deduce from the path of the pole a map of the varying torque which must be acting in order to produce that path, and thence to infer as to the character of the displacements of terrestrial material that must be taking place in order to originate that torque on the Earth as a whole. It has seemed worth while to carry this out in a preliminary way. It has also been thought worth while to set down various dynamical considerations which may prove useful in a systematic analysis of the observational results.

Let ω_1, ω_2, Ω be the component angular velocities of the Earth referred to axes moving with itself, the latter being around the axis of figure. Thus $\omega_1/\Omega, \omega_2/\Omega$ are the angular co-ordinates of the pole of rotation measured on the Earth's surface, and represent

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* Read in part at the British Association, August 1906.